The Idea Behind Krylov Methods

I. C. F. Ipsen

Department of Mathematics
North Carolina State University
Raleigh, NC

ABSTRACT

We explain why it is natural to represent a solution to a linear system as a member of a Krylov space, and why eigenvalues play a central role with regard to existence and uniqueness of solutions determined by Krylov methods.

In particular we show that the solution to a nonsingular linear system $Ax = b$ lies in a Krylov space whose dimension is the degree of the minimal polynomial of A. When the matrix is singular, however, Krylov methods can fail. Even if the linear system does have a solution, it may not lie in a Krylov space. In this case we describe a class of right-hand sides for which a solution lies in a Krylov space. As it happens, there is only a single solution that lies in a Krylov space, and it can be obtained from the Drazin inverse.

This is joint work with Carl Meyer.