RESEARCH STATEMENT

W. GARRETT MITCHENER

1. Specialties

My areas of expertise are dynamical systems, probability, and stochastic processes, with applications to linguistics and biology, particularly to modeling the interaction of learning algorithms, natural selection, and population dynamics.

2. Dissertation: Game dynamics with learning

My dissertation, written under Martin Nowak, was about a game dynamics in the presence of learning. Nowak proposed a generalization of replicator dynamics as a model for evolution of language. In the resulting *language dynamical equation* or *LDE,* a language is analogous to a strategy, but children must learn a language and have a non-zero probability of selecting a different one from their parents. The LDE is parameterized by a payoff matrix and a learning matrix. An extended LDE allows for multiple learning matrices, thereby modeling genetic variation in the language faculty.

If the learning and payoff matrices are highly symmetric, then the dynamics can be completely understood in any number of dimensions: The population converges to either a uniform mixture of all grammars, or to a coherent mixture where one grammar dominates. As learning accuracy increases, a complex sequence of bifurcations takes place.

In an interesting 5 grammar case, each grammar is a Nash equilibrium. Despite the simplicity of the payoff matrix, changing a parameter in the learning matrix causes a period doubling cascade leading to chaos. See Figure 1.

The LDE can exhibit competitive exclusion and coexistence of genetic variants. I prove in a low-dimensional case that if the payoff matrix obeys certain constraints, then competitive exclusion is the only possibility. When this theorem does not apply, the learning process and

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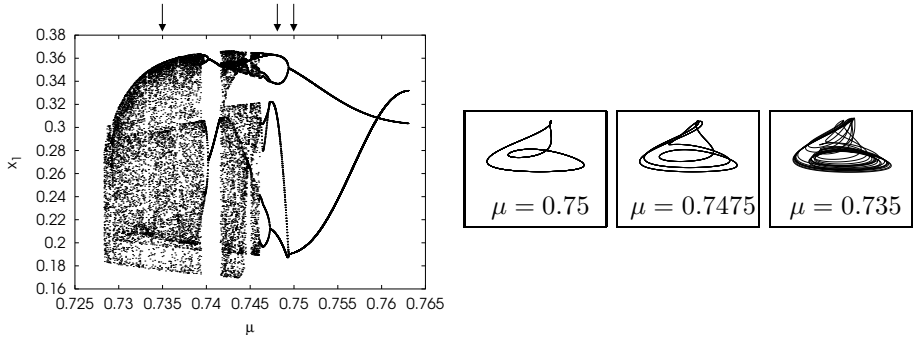

Figure 1. Cascade diagram. The horizontal axis shows a range of values of μ, a parameter in the learning matrix.

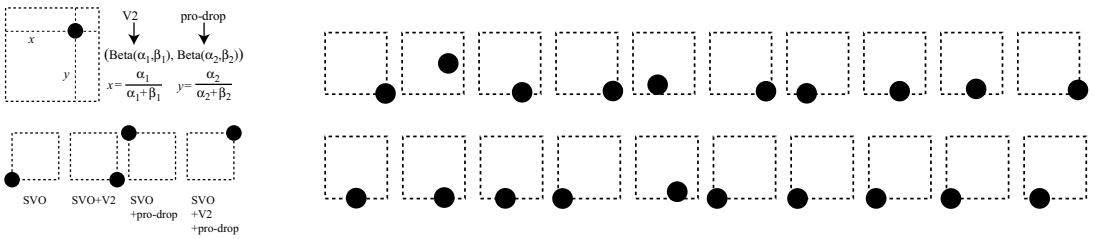

FIGURE 2. Nearly perfect samples from the Markov chain simulation, using two variants of the learning algorithm, for 10 agents configured in a loop.

the history of the population can influence whether an invasion of one variant by another succeeds or fails.

3. Current & future: Variable speech and stochastic processes

My current research focuses on language change on historical rather than evolutionary time scales. This shift in focus enables me to work on problems of more immediate interest to linguists, to work with data, and to develop better models of language acquisition that will be necessary to eventually continue modeling language evolution.

One specific application is a word order change in Middle English, in which two regional dialects effectively merge through increased contact. I began with a dynamical system model for a two compartment population under which a bifurcation results in the extinction of one grammar. This model assumes that each individual uses one idealized grammar, which linguists find unsatisfactory because manuscript data implies that individual speakers use varying mixtures of idealized grammars. I reformulated the model to allow individual speakers to use an arbitrary mixture of the available idealized grammars, resulting in an infinite-dimensional differential equation with similar net behavior: With certain assumptions on the learning process, the mean usage frequencies in each compartment obey the same two-dimensional dynamics as the original model, and the distributions in each compartment converge once the means reach a stable equilibrium. The learning algorithm must have a certain shape so that the population is dominated by one grammar or the other.

Deterministic dynamical systems are limited in their ability to represent processes in perpetual flux, so I have developed a Markov chain model for a discrete agent-based simulation to complement the continuous models. This model contains a detailed learning algorithm, based on Bayesian inference, in which agents take turns trying to parse a sample sentence with a randomly selected idealized grammar. I have implemented a so-called perfect sampling algorithm to draw samples approximately from the chain's stationary distribution. See Figure 2. Strictly speaking, the theorem guaranteeing perfect samples from this algorithm does not apply to this chain. Therefore, I am exploring generalizations to the theorem that would yield a rigorously correct algorithm for this chain.

I have also developed a stochastic differential equation as the limit of an infinite, well-mixed population, which can model a population in constant flux. I plan to study how populations hover around equilibria dominated by a single grammar and shift from one to another.

The most exciting generalization I plan to work on is literacy: The written word allows individuals to learn from the past. Thus, literacy requires dynamics with delay terms, which are notoriously difficult. If X represents the distribution of different speech patterns,

$Q(f)$ is the distribution of children's speech patterns learning from a population distributed according to f, and $\rho(s)$ is the weight children give to speech and literature from time s into the past, then the full delay stochastic dynamics are

$$dX(t) = \beta \left(Q\left(\int_0^\infty X(t-s)\, d\rho(s) \right) - X(t) \right) + \sqrt{X(t)(1-X(t))}\, dW(t) \tag{1}$$

I plan to contribute fundamental results to the theory of non-linear delay ODEs and SDEs in the process of understanding learning dynamics with literacy.

I have applied to the National Science Foundation under Applied Mathematics for a grant to support this research.

4. Collaborations

I am currently working with Misha Becker, a linguist at the University of North Carolina at Chapel Hill. We are developing a model of learning in which statistical patterns provide children with implicit negative evidence. We plan to test it on data concerning particular types of verbs to be collected from parsed corpora of modern English and Middle English. This project could shed light on a major puzzle in linguistics, namely, how might children deduce that certain constructions are ungrammatical given that they never receive explicit information to that effect.

I am also in contact with linguist Anthony Kroch at the University of Pennsylvania concerning word order changes in Middle English and analysis of historical data from parsed manuscripts. Our common interest is the dynamics of language under literacy, and this research could lead to important conclusions on the interpretation of manuscript data.

5. Student research

5.1. **PRUV project with Adam Chandler.** During the summer of 2004, Chandler worked with me through the department's PRUV program to model a sound change spreading through speech in Pennsylvanians. We used a simulation coded in Mathematica, in which a spatially distributed set of agents learned a usage rate subject to invasion by foreigners. We also studied how local interactions lead naturally to reaction-diffusion equations with traveling wave solutions.

Since modeling in linguistics is a fairly new application of mathematics, projects suitable for undergraduates abound. See my teaching statement for more about the modeling class I ran, in which students analyzed language models for their final projects.

5.2. **Toll road problem with Adam Chandler and Pradeep Baliga.** In Spring 2005, Chandler, Baliga, and a third undergraduate, Matthew Mian, competed in the mathematical contest in modeling (MCM). Their paper on the optimum size of a toll plaza won an Outstanding rating. Chandler and Baliga are continuing their work on this problem under my supervision for the 2005-6 academic year. We are studying cellular automaton models of traffic flow, Markov chains, and queuing theory. These students plan to write papers on their work for graduation with distinction in mathematics.

5.3. **Graduate students.** I received an e-mail from a woman named Fang Yi that said she liked my research and asked if I was accepting students. As I am a post-doctoral research associate, I had to decline. I found her interest encouraging.